Stochastic Trends, Economic Fluctuations, and Classical Money Demand Functions: The Evidence from the Different Types of Money *

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Abstract

This paper investigates how the measurement of money affects the empirical support for the balanced-growth framework incorporating money. The analysis tests the framework's three long-run restrictions using real output, consumption, investment, real money balances, and monetary aggregate-specific user costs. I consider four monetary definitions: simple-sum, original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside. Four aggregation levels are used for each Divisia definition and two for simple-sum, yielding 14 monetary aggregates, each paired with its corresponding user cost. Using the U.S. quarterly data through 2026, the Johansen maximum likelihood procedure identifies the long-run relationships and tests the theoretical restrictions. The results show that credit-augmented Divisia aggregates provide more frequent support for the money demand restriction than simple-sum and original Divisia aggregates. The other two restrictions remain unsupported across all four monetary definitions. These findings contribute to the discussion of monetary measurement and the empirical foundations of the balanced-growth framework incorporating money.

JEL classification: C32, E41, E32.
Keywords: Divisia monetary aggregates, money demand, balanced growth hypothesis.

1 Introduction

The balanced growth hypothesis is one of the central implications of macroeconomic theory. It predicts that major macroeconomic aggregates, including real output, consumption, and investment, are driven by a common stochastic trend generated by permanent technological progress. As a result, although these variables may display persistent short-run fluctuations, they should maintain stable long-run relationships consistent with the economy's balanced growth path. The existence and explanatory power of this common stochastic trend therefore provide a direct test of whether the long-run restrictions implied by the theory are supported by the data. If a small number of common stochastic trends explain the long-run behavior of macroeconomic variables, the balanced growth framework provides an empirically consistent account of economic growth and business cycle dynamics. In contrast, if these trends cannot explain the observed long-run behavior of the economy, confidence in the underlying theoretical framework is weakened. Evaluating whether the predicted common stochastic trend captures the long-run behavior of an economy is therefore important not only for assessing the balanced growth hypothesis but also for understanding the role of money in a macroeconomic framework that links real and monetary variables through stable long-run equilibrium relationships.

Despite the strong theoretical foundation of the balanced growth framework King et al. (1988), its empirical performance becomes less convincing when monetary variables are included in the analysis. Empirically, King et al. (1991) show that the common stochastic trend strongly supports the balanced growth restrictions in a three-variable system comprising real output, consumption, and investment. However, adding money supply (M2) and the three-month US Treasury bill rate as the opportunity cost of holding money substantially reduces the explanatory power of the common stochastic trend. This finding raises an important question about the role of money in the economy's long-run structure. If monetary variables are connected to real activity through a stable money demand relationship, as implied by monetary theory, including money should not weaken the empirical support for the theoretical framework. Instead, the decline in explanatory power suggests either that the framework does not fully capture the long-run interactions between real and monetary variables or that the monetary aggregates used in the analysis do not adequately measure the monetary services relevant to economic decisions. Later Serletis and Gogas (2014) attempt to distinguish between these explanations using different monetary aggregates but with same user cost for all aggregates leaving the room of further improvement in the analysis. Therefore it is essential for determining whether the weaker empirical evidence reflects a limitation of the theory or shortcomings in monetary measurement based on appropriate monetary aggregates and user costs.

One possible explanation for the weaker empirical performance of the balanced growth framework is the measurement of monetary aggregates rather than a limitation of the theory itself. Conventional simple-sum monetary aggregates assign equal weights to heterogeneous monetary assets, implicitly treating them as perfect substitutes despite differences in the liquidity and transaction services they provide. (Barnett, 1978, 1980) showed that this approach is inconsistent with economic aggregation theory because it ignores the opportunity cost of holding individual monetary assets. To overcome this limitation, he developed the origi-

nal Divisia monetary aggregates (non-credit-card-augmented). (Barnett and Su, 2016, 2018, 2019) and Barnett et al. (2024) later expanded the concept of monetary services by introducing credit-augmented Divisia monetary aggregates and credit-augmented Divisia inside monetary aggregates. This study compares the empirical performance of simple-sum monetary aggregates with different Divisia aggregates in evaluating the balanced growth framework.

The implications of measuring money using aggregation theory extend well beyond the construction of monetary aggregates. A growing body of evidence ((Schunk, 2001), (Serletis and Rahman, 2013); (Barnett and Park, 2022, 2024, 2025); (Bordo and Duca, 2025); (Ellington, 2018); (Serletis and Xu, 2025); (Chowdhury and Serletis, 2025); etc.) shows that many conclusions based on conventional monetary aggregates change when money is measured using Divisia indexes. The apparent instability of money demand largely reflects financial innovation and changes in the composition of monetary assets rather than a breakdown of the underlying money demand relationship (Lucas and Nicolini, 2015). Likewise, much of the perceived instability in money demand results from monetary mismeasurement (Belongia and Ireland, 2014), while Divisia monetary aggregates and their corresponding user-cost measures provide substantially more information about output, prices, and monetary policy shocks than conventional simple-sum measures (Belongia and Ireland, 2019). Serletis and Gogas (2014) further show that Divisia monetary aggregates satisfy balanced growth restrictions and produce stable long-run money demand functions, whereas comparable simple-sum aggregates often violate the theoretical restrictions implied by economic theory. More recently, Barnett et al. (2022) show that broad Divisia monetary aggregates consistently produce more stable long-run money demand relationships than conventional measures across several advanced and emerging economies. Colunga-Ramos and Valcarcel (2026) also demonstrate that Divisia monetary aggregates improve the identification of monetary policy shocks in an emerging-market economy. Together, this evidence suggests that accurate monetary measurement is essential for drawing reliable inferences about the interaction between money and the macroeconomy.

Although these studies have greatly improved our understanding of monetary measurement, they examine questions different from those addressed in the common stochastic trend literature. King et al. (1988) establish the theoretical foundation for analysing business cycles within a stochastic growth framework, and King et al. (1991) develop the corresponding econometric framework by showing that output, consumption, and investment are driven by a common stochastic trend consistent with balanced growth. Subsequent studies extend this framework in several directions. Farmer (1997) shows theoretically that money can influence macroeconomic dynamics within a real business cycle framework, while Christiano et al. (1999) further document the important role of monetary policy shocks in explaining macroeconomic fluctuations using structural VAR methods. Ahmed and Rogers (2000) examine the long-run effects of inflation by allowing inflation to serve as an additional stochastic trend. Despite these contributions, the role of monetary measurement has received little attention in this strand of the literature.

Serletis and Gogas (2014) provide the closest extension of King et al. (1991) by re-examining the balanced-growth framework using Divisia monetary aggregates. Their findings show that properly measured money provides stronger support for the long-run restrictions implied by balanced-growth theory than conventional simple-sum monetary aggregates. However, important advances in the measure-

ment of monetary services have occurred since their study. First, their analysis is limited to Divisia monetary aggregates detailed in (Barnett et al., 2013) only and does not account for the broader monetary services captured by credit-augmented Divisia monetary aggregates and credit-augmented Divisia inside monetary aggregates. Second, the empirical analysis uses the three-month Treasury bill rate as a common user-cost measure for all monetary aggregates, even though aggregation theory implies that each aggregate should be paired with its own aggregate-specific user cost reflecting the composition of its assets. Finally, the sample predates major changes in monetary policy and financial markets, including the prolonged period of unconventional monetary policy and the post-pandemic inflation episode. These developments motivate a re-examination of whether improved monetary measurement further strengthens empirical support for the common stochastic trend and the long-run restrictions implied by the balanced-growth framework. Motivated by these considerations, this paper revisits the monetary extension of the balanced-growth framework proposed by (King et al., 1991). It examines whether the explanatory power of the common stochastic trend and the associated long-run theoretical restrictions are strengthened when money is measured using aggregate-specific Divisia monetary aggregates and their corresponding aggregate-specific user costs. Along with original Divisia aggregates, the analysis incorporates credit-augmented Divisia monetary aggregates and credit-augmented Divisia inside monetary aggregates, extending the measurement of monetary services to include the transaction services provided by credit. Using an updated sample through 2026, the paper evaluates whether these advances in monetary measurement change the evidence on the long-run relationship between money and real economic activity.

To address these questions, I examine the long-run relationships implied by the balanced-growth framework using the common stochastic trend approach of King et al. (1991). The empirical analysis considers a three-variable system of real output, consumption, and investment, a three-variable money demand system consist of real money balance, output, and user cost, and a five-variable system that incorporates consumption and investment to the three variable money demand system. I compare fourteen monetary aggregates under four definitions of money - simple-sum, original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside - pairing each aggregate with its own user cost. The analysis uses U.S. quarterly data to apply the Johansen maximum likelihood procedure and test the theoretically implied long-run restrictions. The results provide weaker support for the balanced-growth restrictions than that reported by King et al. (1991). More importantly, the common-sample comparison shows that credit-augmented Divisia measures provide more frequent support for the money demand restriction than simple-sum and original Divisia measures in both money-demand systems, although the evidence is not uniform across all augmented aggregates. The consumption-output and investment-output restrictions remain unsupported in systems that include money.

The remainder of the paper is organized as follows. Section 2 presents the theoretical foundations and derives the balanced-growth and money demand restrictions. Section 3 describes the monetary aggregates, data, and graphical patterns. Section 4 outlines the empirical framework. Section 5 presents the cointegration results and tests of the theoretical restrictions. Section 6 concludes.

2 Theoretical Foundations

The theoretical framework is based on King et al. (1991). In the context of a real business cycle model, the single final good, Y_t is produced via a constant-returns-to-scale Cobb-Douglas production function.

$$Y_t = \lambda_t K_t^{1-\theta} L_t^{\theta} \tag{1}$$

where K_t is the predetermined capital stock, chosen in period $t-1$, and L_t is labor input in period t. Total factor productivity, λ_t, follows a logarithmic random walk.

$$\log(\lambda_t) = \mu_\lambda + \log(\lambda_{t-1}) + \xi_t \tag{2}$$

where μ_λ is the average growth rate of productivity and ξ_t is an independent and identically distributed process with mean zero and variance σ^2. In equation (2), $\mu_\lambda + \log(\lambda_{t-1})$ represents the deterministic part of the productivity evolution and ξ_t represents the stochastic innovations (or shocks). This specification implies that productivity follows a random walk with drift and generates a stochastic trend in real economic activity.

Within this framework, consumption, investment, and output grow along a balanced growth path at a rate determined by technological progress. In the deterministic version of the model, the steady-state growth rate of the economy equals the rate of productivity growth. When productivity follows a stochastic trend, the logarithms of consumption, investment, and output inherit the same stochastic component. Let $X_t = [c_t, i_t, y_t]$ denote the logarithms of real per capita consumption (c_t), investment (i_t), and output (y_t). Because productivity follows a random walk, each element of X_t is integrated of order one [or $I(1)$ in the terminology of (Engle and Granger, 1987)]. However, balanced growth restrictions imply that certain linear combinations of these variables are stationary. In particular, the great ratios $c_t - y_t$ and $i_t - y_t$ should be stationary in the long run. These restrictions imply that consumption, investment, and output share a common stochastic trend and therefore are cointegrated. The presence of these long-run relationships provides the theoretical basis for empirical tests using cointegrated vector autoregressive models. Two cointegrating vectors, [1,0,-1] and [0,1,-1], can represent the great ratios, respectively.

The framework can also be extended to incorporate monetary variables. In particular, X_t can be augmented to include real per capita money balances, $(m-p)_t$, and the nominal interest rate/user costs, UC_t, that is, $X_t = [c_t, i_t, (m-p)_t, y_t, UC_t]$. Then, if $(m-p)_t$ and R_t are each integrated of order one, according to the theory we would expect to find three cointegrating vectors the two great ratios, [1,0,0,-1,0] and [0,1,0,-1,0], and the money demand relation, $[0, 0, 1, \beta_y, \beta_R]$. In fact, according to the theory we expect $\beta_y = -1$ and β_R to be small and positive. These coefficients in the cointegrating vector imply a one-to-one positive relation between real money balances and real output and a small but negative relation between real balances and the nominal rate of interest. King et al. (1991) find a common stochastic trend in the three variable system incorporating output, consumption and investment for the U.S. data. But the common stochastic trend loses explanatory power significantly when money balances and the nominal interest rate are added to the system.

3 Data

3.1 Alternative Measures of Money

As this paper examines whether monetary measurement affects the empirical support for the common stochastic trend implied by the monetary extension of the balanced-growth framework, the analysis considers four classes of monetary aggregates that differ in their theoretical construction. These measures range from conventional simple-sum aggregates to the latest Divisia aggregates that incorporate both monetary assets and credit card transaction services. Comparing these measures allows us to determine whether the empirical evidence depends on the measurement of monetary services rather than on the underlying theoretical restrictions.

The first class consists of the conventional simple-sum monetary aggregates, Sum M1 and Sum M2. These measures are constructed by adding the outstanding quantities of their component monetary assets with equal weights. This procedure implicitly assumes that all monetary assets are perfect substitutes and provide identical monetary services per dollar held. Although simple-sum aggregates remain widely used in empirical macroeconomics and monetary policy analysis, (Barnett, 1978, 1980) showed that this weighting scheme is inconsistent with economic aggregation theory because it ignores differences in the opportunity cost of holding individual monetary assets. As a result, these aggregates may contain substantial measurement error when their component assets provide different degrees of liquidity.

To address this limitation, Barnett (1980) developed the Divisia monetary aggregates, which measure money as a weighted sum rather than a simple sum of financial assets. Instead of assigning equal weights, Divisia indexes weight each component by its user cost, reflecting the value of the monetary services it provides relative to a benchmark asset. Assets providing greater transaction services receive larger weights, whereas those serving mainly as stores of value receive smaller weights. Because these weights vary over time with interest rates and portfolio composition, Divisia aggregates provide a theoretically consistent measure of money based on aggregation theory. The empirical analysis considers Divisia aggregates at the M1, M2, M3, and M4 levels, allowing comparisons across progressively broader definitions of money.

Recent advances in monetary aggregation extend this framework by recognizing that households and firms obtain transaction services from both monetary assets and credit cards. Barnett et al. (2024) incorporated these deferred-payment transaction services into the Divisia framework through credit-augmented Divisia monetary aggregates. These measures jointly aggregate the monetary services generated by conventional monetary assets and credit cards using aggregation-theoretic user-cost weights. Barnett and Su (2018) also developed credit-augmented Divisia inside monetary aggregates, which incorporates financial intermediaries by focusing on privately supplied inside money and credit card services while excluding outside money, such as currency and Treasury bills. Together, these measures provide a theoretically superior and complete representation of monetary services currently available and allow us to assess whether a superior definition of money strengthens empirical support for the common stochastic trend. The theoretical construction of the monetary aggregates is summarized in Appendix A.

3.2 Data Sources and Variable Construction

I use data from multiple sources because no single source provides all variables required for the analysis. The two main data sources of the paper are Federal Reserve Economic Data (FRED) and the Center for Financial Stability (CFS). CFS provides the Divisia, credit-augmented Divisia, and credit-augmented Divisia inside monetary aggregates, along with their corresponding user costs. FRED is the source of all other macroeconomic data. These data include Real Personal Consumption Expenditure, Real Gross Private Domestic Investment, Real Gross National Product, Real Government Consumption Expenditures and Gross Investment, Real State & Local Consumption Expenditures & Gross Investment, population, GNP deflator, simple-sum M1, and simple-sum M2.

I use quarterly data over the sample period from 1967:q1 to 2026:q2. Since our objective is to examine outcomes related to business cycle models, quarterly data are appropriate because they align with the conventional frequency used in business cycle analysis. The starting point of our sample is constrained by the availability of Divisia monetary aggregates. In addition, a shorter sample period, from 2006:q3 to 2026:q2, is used to examine the credit-augmented Divisia and credit-augmented Divisia inside monetary aggregates, as data for these measures are available only from 2006:q3 onward.

Among the collected data, the Real Private Fixed Investment series and the Real State & Local Consumption Expenditures & Gross Investment series are not available in the same base year over the full sample period. The Fixed Investment series is available from 2007:q1 to 2026:q2 in 2017 dollars, while earlier observations are reported in 2009 dollars. To maintain consistency, I convert the earlier series into 2017 dollars through backward splicing, following the approaches used in Bulman and Simon (2003), Muller et al. (2025), and the World Bank Data Help Desk (nd).[1] To splice the series backward, the process starts by computing the ratio between the two base-year series for the latest common quarter. The the ratio is applied to convert the observations reported in the older base year into the newer base year. I follow the same procedure for the Real State & Local Consumption Expenditures & Gross Investment series, which is also reported in 2017 dollars from 2007:q1 to 2026:q2, while earlier observations are available in 2009 dollars. Therefore, the earlier portion of the series is converted into 2017 dollars through backward splicing.

The final dataset ensures comparability with earlier studies, including Serletis and Gogas (2014) and King et al. (1991). To construct logged real per capita consumption, I divide real consumption expenditure by population and take the logarithm of the resulting series. Likewise, for logged real per capita investment, real private fixed investment is divided by population followed by logarithmic transformation. To derive logged real per capita output, I first construct Real Private GNP by deducting Real State & Local Consumption Expenditures & Gross Investment from Real GNP, following Serletis and Gogas (2014). Then I divide Real Private GNP by population and take the logarithm of the resulting series. In addition, monetary aggregates are adjusted by the GNP deflator and population to obtain real per capita money balances. Later I take the logarithm of these series to obtain logged real money balances per capita $(m_t - p_t)$. The empirical analysis

[1] https://datahelpdesk.worldbank.org/knowledgebase/articles/114946-how-can-i-rescale-a-series-to-a-different-base-yea

uses the user cost series for each monetary aggregate provided by CFS. For Sum M1 and Sum M2, I derive the corresponding actual user costs using the Leontief index by taking the minimum user cost among the their respective component assets. Here, the actual user cost refers to the benchmark user cost applied to the simple-sum aggregate based on the minimum user cost across its underlying assets. Thus, the final dataset for my empirical analysis gets ready.

3.3 Descriptive Analysis

Since this study uses quarterly time-series data, the investigation begins with a graphical review of the variables. Figures 1–10 summarize the data used in the study. Figure 1 plots the logged real per capita flow variables: private output, personal consumption expenditure, and private fixed investment. All three variables display upward trends, although their variability differs over time. The logged real per capita private fixed investment appears substantially more volatile than the other two variables. These patterns suggest that the series may not be stationary at level, as their means and variance change over time. However, balanced growth theory predicts that linear combinations of consumption and output, and investment and output, should be stationary. Therefore I plot the consumption-to-output and investment-to-output great ratios in Figure 2. The figure shows that both ratios fluctuate over the sample period from 1967:q1 to 2026:q2. The investment-output ratio exhibits larger fluctuations than the consumption-output ratio, particularly in the later part of the sample. Both ratios also show changes in their levels over time. Overall, the graphical evidence does not clearly establish stationarity of the great ratios, which is examined further through the empirical framework.

Figure 3 plots the logged real per capita M1 monetary aggregates under simple-sum, original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside measures of money. To allow meaningful comparison across measures, all M1 aggregates are normalized to one. The series display declining trends over much of the sample, although their movements differ across measures. The original Divisia M1 measure remains above the simple-sum M1 measure for much of the period, before the two series converge in the later years. The simple-sum measure also exhibits several noticeable declines. Figure 4 presents a similar plot for the M2 level of aggregation. The simple-sum M2 measure declines substantially over the sample, while the original Divisia M2 measure remains relatively stable during much of the earlier period before declining in later years. The two measures converge around the middle of the 2000s, after which the simple-sum measure declines more sharply. Figures 5 and 6 plot the M3 and M4 levels of aggregation, respectively, for the original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside measures. Both figures show an increase in the original Divisia measures during the earlier part of the sample, followed by a gradual decline that becomes more pronounced from the late 1990s onward. The credit-augmented measures broadly follow the original Divisia measures over their available periods, although their subsequent movements differ. Overall, these patterns suggest persistent movements in the monetary aggregates requiring further examination using formal time-series tests.

Figures 7 to 10 plot the user costs of the monetary aggregates used in the analysis. Figure 7 reports the user costs of M1 aggregates under simple-sum,

original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside methods of monetary aggregation. Among the four measures, the user cost of Sum M1 appears more volatile than that of Divisia M1, with several sharp increases and declines throughout the sample. Its largest fluctuations occur during the earlier part of the sample, although substantial movements also occur in later years. By contrast, the user cost of Divisia M1 remains within a narrower range. The credit-augmented Divisia M1A and M1AI user costs are available only for the later part of the sample. Both decline sharply near the beginning of their displayed periods and subsequently remain at relatively low levels. Figure 8 plots the user costs for all four measures at the M2 level of aggregation. Both the simple-sum and original Divisia M2 user costs exhibit considerable variation, with pronounced increases during the early 1980s and again in the later part of the sample. The two series experience a sharp decline near the end of the period shown, followed by different patterns of recovery. The credit-augmented M2A and M2AI user costs also decline sharply at the beginning of their available periods and subsequently remain below their earlier peaks.

Figures 9 and 10 present the user costs for the M3 and M4 levels of aggregation, respectively, for the original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside monetary aggregates. In both figures, the original Divisia user costs exhibit substantial fluctuations, including sharp increases during the early 1980s and renewed increases in the later part of the sample. Both series then decline sharply near the beginning of the period covered by the credit-augmented measures. The M3A and M4A user costs subsequently remain above their corresponding inside measures, M3AI and M4AI, and the original Divisia measures. Overall, Figures 7–10 show considerable variation in user costs across monetary aggregation methods. The distinct movements and changes in levels suggest that the time-series properties of these measures warrant further investigation. I therefore examine their stationarity formally in the following analysis.

4 Empirical Framework

I use the Johansen maximum likelihood procedure (hereafter, the Johansen cointegration test) to examine long-run equilibrium relationships in a multivariate system following (Johansen, 1988) and (Johansen and Juselius, 1990). The lag length of the underlying vector autoregressive (VAR) model is selected using the Schwarz Information Criterion (SIC), subject to residual diagnostic tests. Whenever necessary, I adjust the SIC-selected lag length to address serial correlation. The resulting lag order is then used to estimate the vector error correction model (VECM). The trace (λ_{trace}) and maximum eigenvalue ($\lambda_{\max}$) test statistics help identify the number of cointegrating vector(s). The null hypothesis of the trace test posits the existence of at most r cointegrating vectors and is evaluated against a general alternative hypothesis. On the other hand, the maximum eigenvalue test considers a specific alternative hypothesis. The corresponding p-values and critical values are based on the response surface approximations developed by (MacKinnon et al., 1999). I aim to examine whether the rejection of the balanced growth hypothesis and classical money demand theory by King et al. (1991) relates to the use of the proper user cost of money and augmented Divisia monetary aggregates.

The Johansen procedure begins with the p-dimensional VAR model,

$$X_t = \mu_0 + \mu_1 t + \sum_{i=1}^{k} \Phi_i X_{t-i} + \varepsilon_t \tag{3}$$

where X_t denotes the $p \times 1$ vector of endogenous variables, μ_0 and μ_1 are vectors of intercepts and deterministic trend coefficients, respectively, Φ_i denotes the coefficient matrix associated with lag i, k is the lag length, and ε_t is a vector of independently and identically distributed innovations. Since the variables considered in this study are integrated of order one, the VAR can be rewritten in the VECM form,

$$\Delta X_t = \mu + \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \tag{4}$$

where

$$\Pi = \alpha\beta' \tag{5}$$

The matrix β contains the cointegrating vectors that characterize the long-run equilibrium relationships among the variables, while the matrix α contains the corresponding loading coefficients that measure the speed at which each variable adjusts to deviations from the long-run equilibrium. The rank of Π determines the number of linearly independent cointegrating vectors and is identified using the trace (λ_{trace}) and maximum eigenvalue ($\lambda_{\max}$) statistics. Once the cointegration rank has been established, economically meaningful long-run relationships can be identified by imposing theory-based restrictions on the cointegrating vectors contained in β. These restrictions are evaluated within the VECM framework using the likelihood-ratio (LR) test for overidentifying restrictions, which is asymptotically distributed as a chi-square random variable under the null hypothesis. To be consistent with the implications of the balanced-growth framework and the long-run money demand relationship developed in the theoretical section, my empirical analysis adopt the unrestricted constant specification (Case 3b) to allow for non-zero intercepts in the short-run dynamics while excluding deterministic linear trends in the cointegrating relationships. This specification is consistent with the assumption that the variables may contain stochastic trends but their long-run relationships do not exhibit deterministic linear trends.

Prior to implementing the cointegration analysis, I assess the unit root properties of the variables included in the model to verify that they enter the cointegration framework at the same order of integration, as suggested by the graphical analysis. To this end, this paper employs the Augmented Dickey-Fuller (ADF) test [see (Dickey and Fuller, 1981)] and the Dickey-Fuller GLS (DF-GLS) test [see (Elliott et al., 1996)], incorporating both a constant and a trend in the model specifications. For both tests, the optimal lag length is also selected using the SIC. Since the ADF and DF-GLS tests exhibit limited power against trend-stationary alternatives, I additionally report results from the KPSS test [see (Kwiatkowski et al., 1992)], which tests the null hypothesis of stationarity around a constant and trend against the alternative of a unit root. The combined evidence from these three tests determine whether the variables share a common order of integration.

The empirical investigation in this paper considers two models. One is a three-variable approach, while the other is a multivariate approach that includes five variables. For the three-variable approach, one variant uses logged real consumption per capita (c_t), logged real investment per capita (i_t), and logged real output per capita (y_t) to examine the balanced growth implication of the growth model outlined in the theoretical section. This approach extends King et al. (1991) by incorporating recent economic dynamics through an extended sample period. Theory suggests two cointegrating vectors for the system (c_t, i_t, y_t) because the differences $(c_t - y_t)$ and $(i_t - y_t)$ should become stationary processes.

A second variant of the three-variable approach builds a system that includes logged real money balance per capita $(m_t - p_t)$, logged real output per capita (y_t), and the actual user cost of holding money (UC_t) in levels. Theory suggests one cointegrating vector for this system, $(m_t - p_t, y_t, UC_t)$, which represents the long-run money demand function. For this system, I use fourteen monetary aggregates under four definitions: simple-sum, Divisia, credit-augmented Divisia, and credit-augmented Divisia inside money. The two available simple-sum aggregates are Sum M1 and Sum M2. For the other three monetary definitions, four levels of aggregation available from CFS are considered. The Divisia aggregates include Divisia M1, Divisia M2, Divisia M3, and Divisia M4. The credit-augmented Divisia aggregates comprise Divisia M1A, Divisia M2A, Divisia M3A, and Divisia M4A. Similarly, the credit-augmented Divisia inside monetary aggregates include Divisia M1AI, Divisia M2AI, Divisia M3AI, and Divisia M4AI. This three-variable system differs from both King et al. (1991), which uses only one monetary aggregate, and Serletis and Gogas (2014), which examines ten monetary aggregates, excluding the recently developed augmented Divisia aggregates, while applying the same user cost to all aggregates. Hence, the three-variable money demand system examined in this paper extends both studies.

I then move to the multivariate approach by adding logged real consumption per capita and logged real investment per capita to the three-variable money demand system. As discussed in the theoretical framework, the theory suggests three cointegrating vectors for the five-variable system, $(c_t, i_t, m_t - p_t, y_t, UC_t)$. One vector captures the consumption-output ratio with the structure [1, 0, 0, -1, 0]. Another captures the investment-output ratio with the structure [0, 1, 0, -1, 0]. The final vector represents the money demand function with the structure [0, 0, 1, -1, β_{UC}].

After identifying the number of cointegrating vectors for each system and monetary aggregate using the Johansen cointegration test, the theoretical interpretation of each cointegrating vector is examined. Because the cointegration test alone does not provide a clear identification of the theoretical vector(s), I explore overidentifying restrictions through the related VECM. Each cointegrating vector identified by the cointegration test is tested separately. In each case, the restriction is tested under the null hypothesis using the Wald statistic, which follows a chi-square distribution. The empirical analysis is conducted using EViews 13.

5 Empirical Evidence

5.1 Long-Run Cointegration Results

I first examine the unit-root properties of the variables within the empirical framework of this paper. Tables 1, 4, and 5 report the results for the real flow variables, logged real per capita monetary aggregates, and user costs, respectively. Following the decision rule described in the empirical framework, a variable is classified as $I(1)$ when at least two of the three tests indicate a unit root at level. The results classify the real flow variables and all logged real per capita monetary aggregates as $I(1)$. Among the user costs, only the user cost of Sum M1 is classified as $I(0)$; the remaining user costs are classified as $I(1)$. The first-difference results are reported in Appendix Tables A1-A3. The unit-root findings provide the basis for the subsequent cointegration analysis.

We begin with the three-variable system $\{c_t, i_t, y_t\}$ over 1967:q1-2026:q2. Table 2 identifies one cointegrating vector, although balanced-growth theory predicts two, corresponding to the consumption-output and investment-output ratios. Table 3 reports the tests of these restrictions. Both the consumption-output restriction $[1, 0, -1]$ and the investment-output restriction $[0, 1, -1]$ are rejected at the 5% level. This differs from King et al. (1991), who found support for both restrictions, and indicates weaker empirical support for balanced growth over the extended sample. The difference may reflect changes in economic structure over the longer period, although the present analysis does not directly test for structural instability.

We next examine the three-variable money demand system $\{(m_t - p_t), y_t, UC_t\}$ using the simple-sum and original Divisia aggregates over 1967:q1-2026:q2. Tables 6 and 7 report the cointegration results and tests of the money demand restriction. Table 6 identifies one cointegrating vector only for Divisia M2; no vector is identified for Sum M2 or Divisia M1, M3, and M4. Table 7 shows that the money demand restriction is rejected for Divisia M2 at the 5% level. Thus, none of the simple-sum or original Divisia aggregates provides evidence supporting the restricted money demand relationship over the full sample. This differs from Serletis and Gogas (2014), although the comparison should be treated with caution because they use the Treasury bill rate as a common opportunity-cost measure, whereas this paper uses the corresponding user cost for each aggregate. Differences in sample period and aggregate definitions may also contribute to the different findings.

We then add consumption and investment to form the five-variable system $\{c_t, i_t, (m_t - p_t), y_t, UC_t\}$. Tables 8 and 9 report the cointegration tests and the tests of the theoretical restrictions for the simple-sum and original Divisia aggregates over the full sample. Table 8 identifies at least one cointegrating vector for each aggregate, with two vectors for Divisia M2. However, Table 9 shows that the restrictions for the consumption-output ratio, investment-output ratio, and money demand function are rejected at the 5% level across all specifications. The estimated user-cost coefficients are positive, consistent with theory. However, this sign alone does not establish support for the money demand restriction. Therefore, the five-variable results do not support the full set of theoretical restrictions for the simple-sum and original Divisia aggregates.

We next examine the credit-augmented Divisia and credit-augmented Divisia inside aggregates over their available sample, 2006:q3-2026:q2. Table 10 shows that

the three-variable money demand system has one cointegrating vector for Divisia M1A, M2A, M3A, M4A, M1AI, and M2AI, but none for Divisia M3AI or M4AI. As a cointegrating vector does not necessarily satisfy the theoretical money demand restriction, Table 11 reports the restriction tests for the six identified vectors. The restriction is not rejected at the 5% level for Divisia M2A ($p = 0.677$), Divisia M3A ($p = 0.274$), and Divisia M2AI ($p = 0.728$). It is rejected for Divisia M1A ($p = 0.008$), Divisia M4A ($p = 0.038$), and Divisia M1AI ($p = 0.006$). Divisia M4A is not rejected at the 1% level, but it is rejected under the main 5% criterion. These results provide support for the money demand restriction for three of the six augmented aggregates with an identified vector, a more favorable result than that obtained for the simple-sum and original Divisia aggregates in the full-sample three-variable system.

We then examine the five-variable system using the same eight credit-augmented aggregates. Table 12 identifies two cointegrating vectors for each aggregate. Table 13 shows that the restrictions corresponding to the consumption-output and investment-output ratios are rejected across all eight specifications. By contrast, the money demand restriction is not rejected at the 5% level for Divisia M1A, M2A, M3A, M4A, M1AI, M2AI, and M3AI. It is rejected for Divisia M4AI ($p = 0.032$), although it is not rejected at the 1% level. The five-variable system therefore supports the money demand restriction for seven of the eight augmented aggregates, but it does not support the full set of balanced-growth restrictions. The results suggest that credit augmentation improves the empirical support for the money demand relationship, particularly in the five-variable system, while the great-ratio restrictions remain unsupported.

In summary, the results show that the empirical support for the balanced-growth framework depends on the measurement of monetary aggregates. While the extended sample provides weaker support for the balanced-growth restrictions than that reported by King et al. (1991), neither simple-sum nor original Divisia aggregates supports the restricted money demand relationship. Credit-augmented Divisia aggregates, however, provide stronger evidence for money demand, particularly in the five-variable system, where seven of the eight aggregates satisfy the restriction at the 5% level. Nevertheless, the consumption-output and investment-output ratio restrictions remain unsupported across all five-variable specifications. Overall, these findings highlight the importance of monetary measurement in assessing the long-run role of money within the balanced-growth framework, while showing that improved monetary measurement alone does not restore support for the full set of theoretical restrictions.

5.2 Robustness

The different sample periods for the original and credit-augmented measures make a common-sample comparison important. The analysis above uses 1967:q1-2026:q2 for the simple-sum and original Divisia aggregates and 2006:q3-2026:q2 for the augmented aggregates. Therefore I re-estimate the systems for the simple-sum and original Divisia aggregates over the common sample, 2006:q3-2026:q2, to assess whether the differences in results may be explained by sample coverage.

Tables 14 and 15 report the common-sample results for the three-variable money demand system. Table 14 identifies one cointegrating vector only for Divisia M1; no vector is identified for Sum M2 or Divisia M2, M3, and M4. Table 15 shows

that the money demand restriction is rejected for Divisia M1 at the 5% level. Thus, none of the simple-sum or original Divisia aggregates supports the restricted money demand function in the common-sample three-variable system.

Tables 16 and 17 report the corresponding results for the five-variable system. Table 16 identifies at least one cointegrating vector for each simple-sum and original Divisia aggregate, with two vectors for Divisia M4. Table 17 shows that the money demand restriction is rejected for Sum M2 and Divisia M1, M2, and M3, but is not rejected for Divisia M4 ($p = 0.146$), whose estimated user-cost coefficient is positive. The consumption-output and investment-output restrictions are rejected for all five aggregates. The evidence for the original measures is therefore limited: only Divisia M4 supports the money demand restriction in the common-sample five-variable system, and none supports the full set of theoretical restrictions.

Taken together, the common-sample comparison shows that none of the simple-sum or original Divisia aggregates supports the money demand restriction in the three-variable system, while only Divisia M4 does so in the five-variable system. In contrast, credit-augmented Divisia aggregates provide stronger support for the money demand function in both three-variable and five-variable systems. These results do not establish the full set of balanced-growth restrictions, since the consumption-output and investment-output restrictions remain rejected in systems that include money. They nevertheless suggest that credit-augmented Divisia monetary aggregates improve empirical support for the theoretical money demand relationship. Improved monetary measurement can therefore strengthen evidence of theoretical consistency in macroeconomic data, even when the broader balanced-growth restrictions remain unsupported.

6 Conclusion

This paper re-examines the monetary extension of the balanced-growth framework, focusing on whether the empirical evidence for a common stochastic trend varies with the choice of monetary aggregate. Building on King et al. (1991), I analyse the long-run relationships among real output, consumption, investment, real money balances, and the user cost of money using four types of monetary aggregates: simple-sum, original Divisia, credit-augmented Divisia, and credit-augmented Divisia inside measures. The analysis considers both three-variable and five-variable systems, with each monetary aggregate matched to its corresponding user cost in accordance with aggregation theory. Applying the Johansen maximum likelihood procedure and testing the implied long-run restrictions, I find that the empirical validity of the monetary extension depends on how monetary aggregates are measured. The simple-sum and original Divisia aggregates under longer sample offers less support for the balanced-growth restrictions than that found by King et al. (1991), with the restrictions linking consumption and investment to output generally rejected. However, credit-augmented Divisia measures perform better in tests of the long-run money demand restriction than simple-sum and original Divisia aggregates. This difference is evident in the common sample: none of the simple-sum or original Divisia aggregates satisfies the money demand restriction in the three-variable system, while only Divisia M4 does so in the five-variable system. By comparison, the restriction is not rejected for three of the six credit-augmented aggregates with an identified cointegrating relationship in the

three-variable system and seven of the eight aggregates in the five-variable system. These findings demonstrate that the choice of monetary aggregate matters when assessing whether long-run macroeconomic relationships are consistent with monetary theory.

The findings have several implications for macroeconomic theory and empirical monetary research. First, the limited evidence for the complete set of balanced-growth restrictions indicates that the monetary extension of the framework receives only partial empirical support, even when money is measured using aggregation-theoretic indexes. Second, the stronger performance of credit-augmented Divisia measures in testing the money demand restriction shows that monetary measurement can influence conclusions about long-run economic relationships. Third, including credit-related services in Divisia aggregates appears to improve the empirical identification of long-run money demand. This improvement, however, is not common to all monetary aggregates and does not extend to the restrictions relating consumption and investment to output. Taken together, the results reinforce the value of developing and applying monetary aggregates based on economic aggregation theory in empirical macroeconomics. Further work on balanced growth, monetary business cycle models, and monetary policy transmission may gain from measures that account more fully for the monetary services supplied by both financial assets and credit instruments.

One limitation of this study is that it does not examine possible structural instability over the sample period 1967:q1 - 2026:q2, which includes substantial changes in monetary policy and financial markets. Recent studies provide relevant evidence on this issue. Movaghari et al. (2026), using copula methods to assess the stability of money demand, find no evidence that original Divisia aggregates outperform simple-sum measures. Similarly, Chowdhury and Serletis (2025) find no evidence of stable money demand for original Divisia aggregates when applying the (Pesaran et al., 2001) bounds testing procedure. However, their analysis of the post-2006 period, for which credit-augmented Divisia aggregates are available, finds stable money demand relationships using both original and credit-augmented Divisia measures.

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Figure 1: Logged Real per Capita Flow Variables

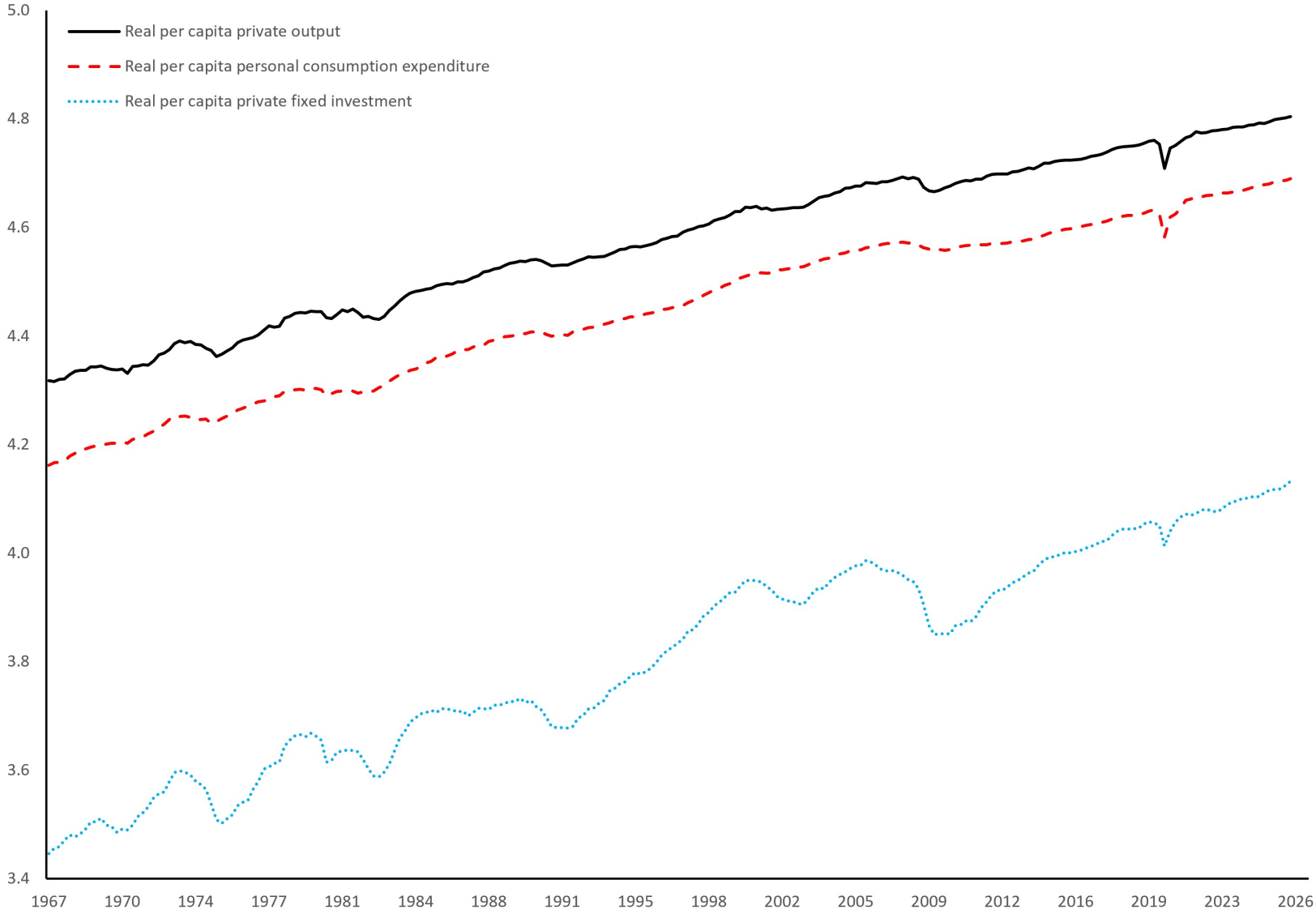


Figure 2: The Great Ratios

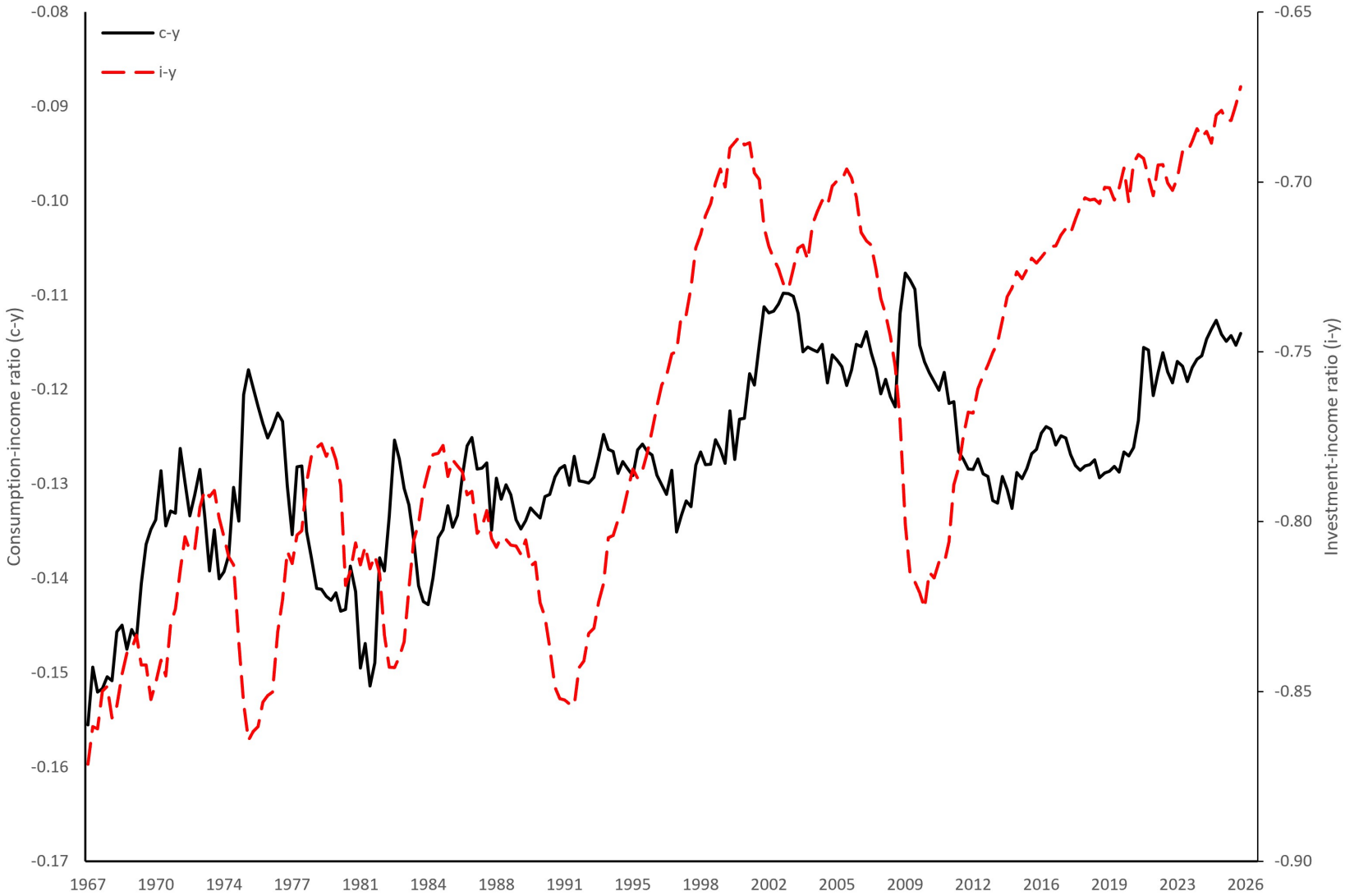
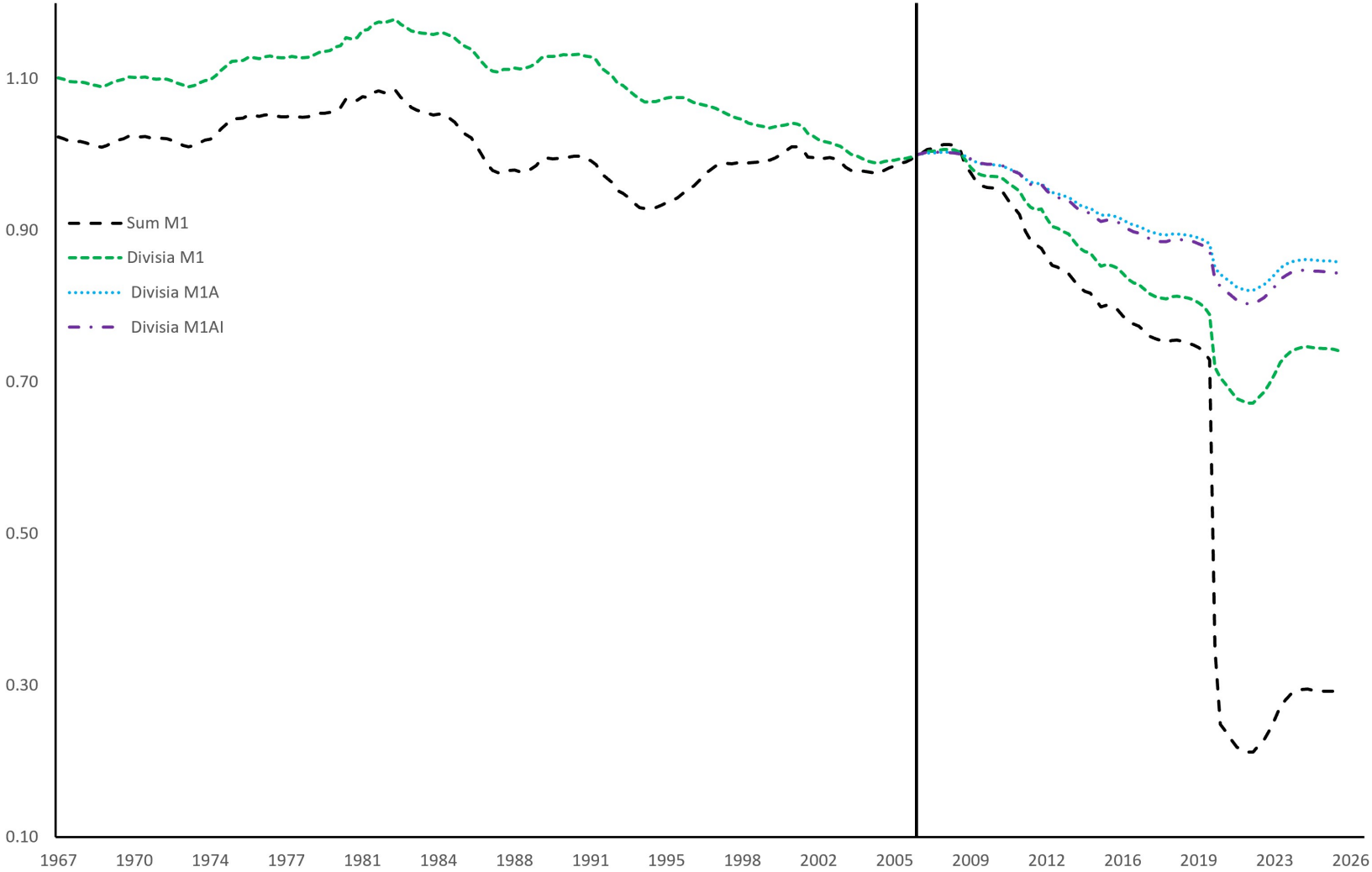


Figure 3: Logged Real per Capita M1 Aggregates



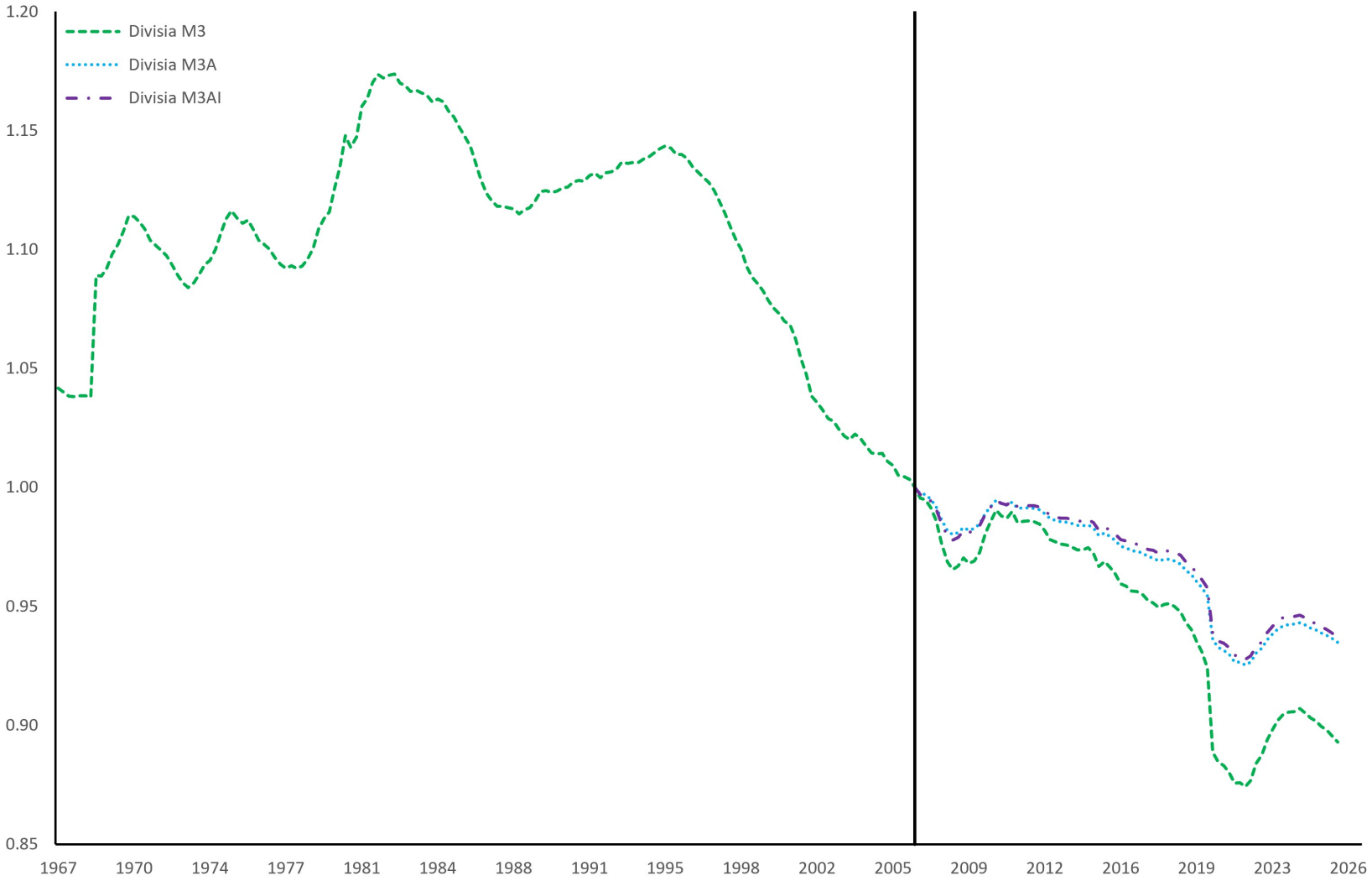
Note: The vertical line indicates the quarter (2006:q3) at which the logged real per capita aggregates were normalized to 1. This is also when the credit-augmented aggregates were introduced.

Figure 4: Logged Real per Capita M2 Aggregates



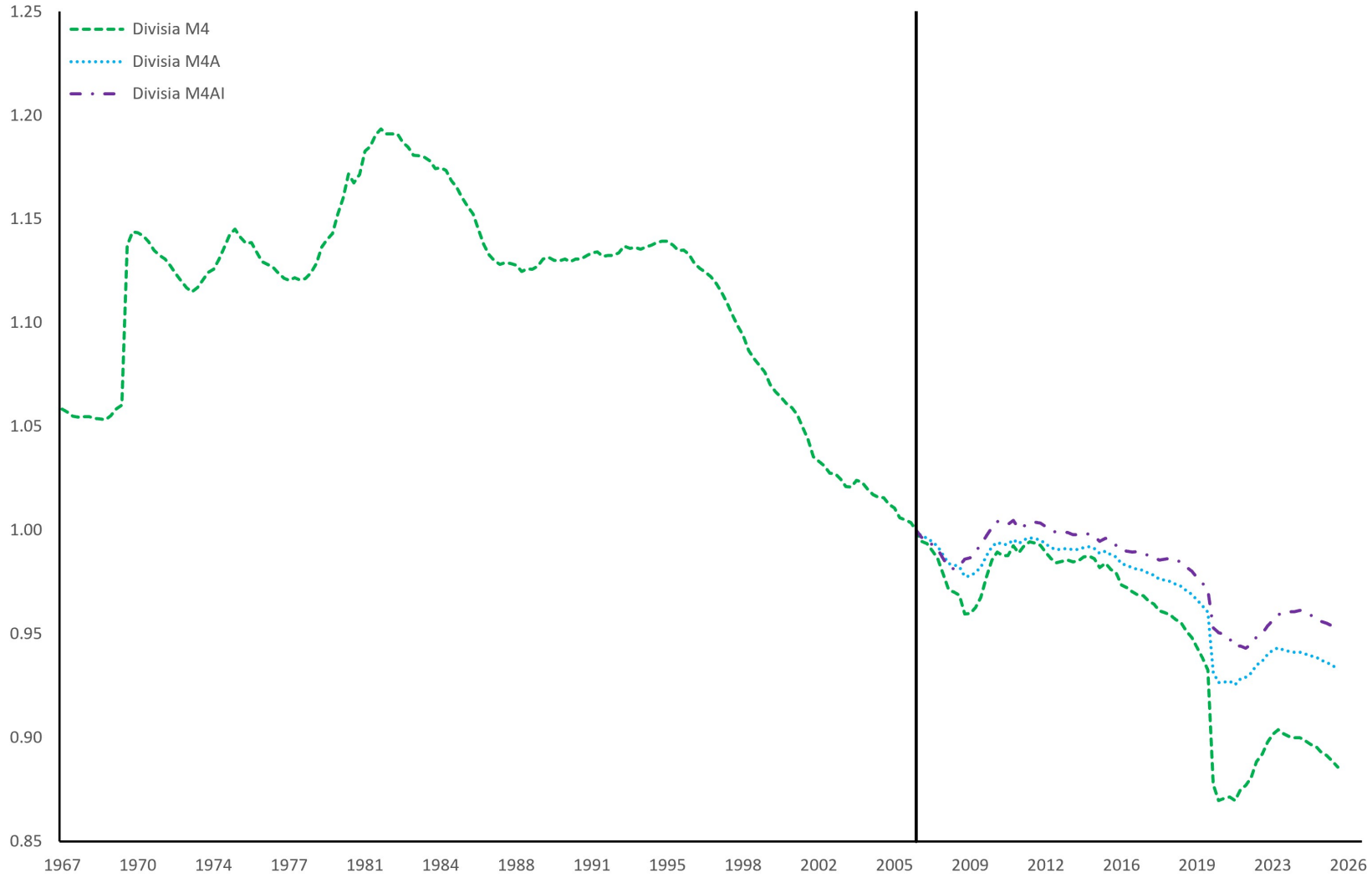
Note: The vertical line indicates the quarter (2006:q3) at which the logged real per capita aggregates were normalized to 1. This is also when the credit-augmented aggregates were introduced.

Figure 5: Logged Real per Capita M3 Aggregates



Note: The vertical line indicates the quarter (2006:q3) at which the logged real per capita aggregates were normalized to 1. This is also when the credit-augmented aggregates were introduced.

Figure 6: Logged Real per Capita M4 Aggregates



Note: The vertical line indicates the quarter (2006:q3) at which the logged real per capita aggregates were normalized to 1. This is also when the credit-augmented aggregates were introduced.

Figure 7: The User Costs of the M1 Aggregates

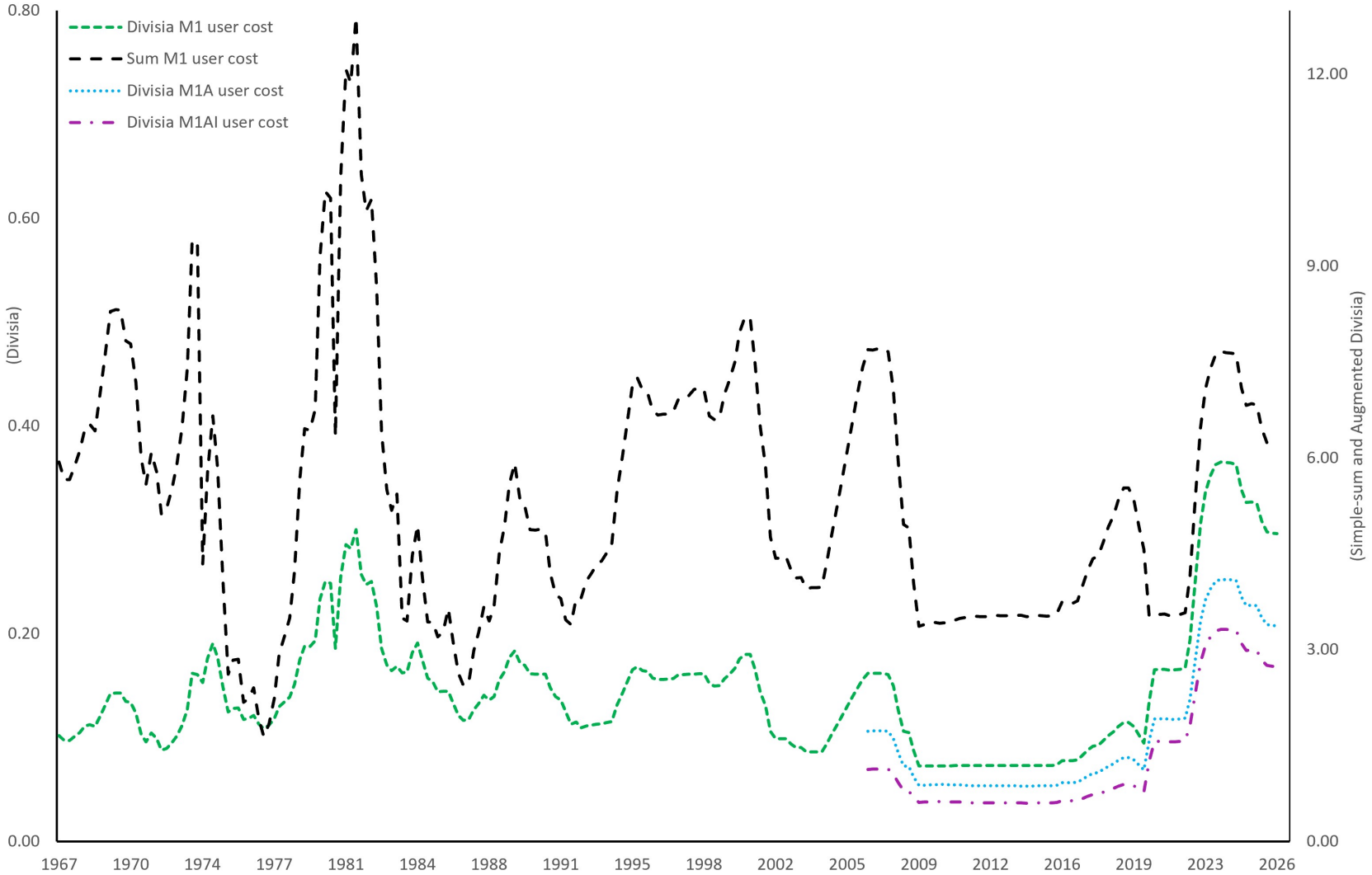


Figure 8: The User Costs of the M2 Aggregates



Figure 9: The User Costs of the M3 Aggregates

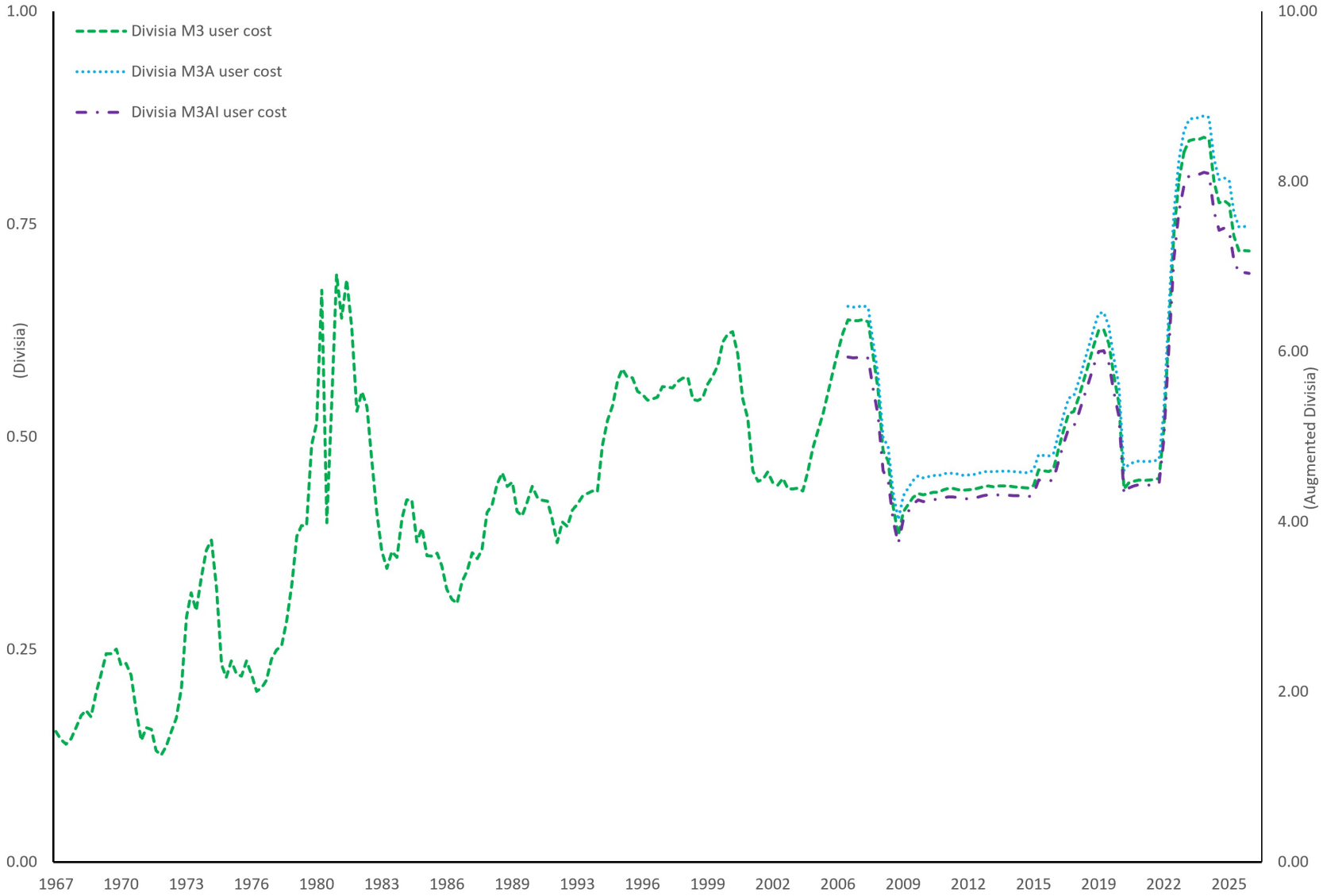


Figure 10: The User Costs of the M4 Aggregates

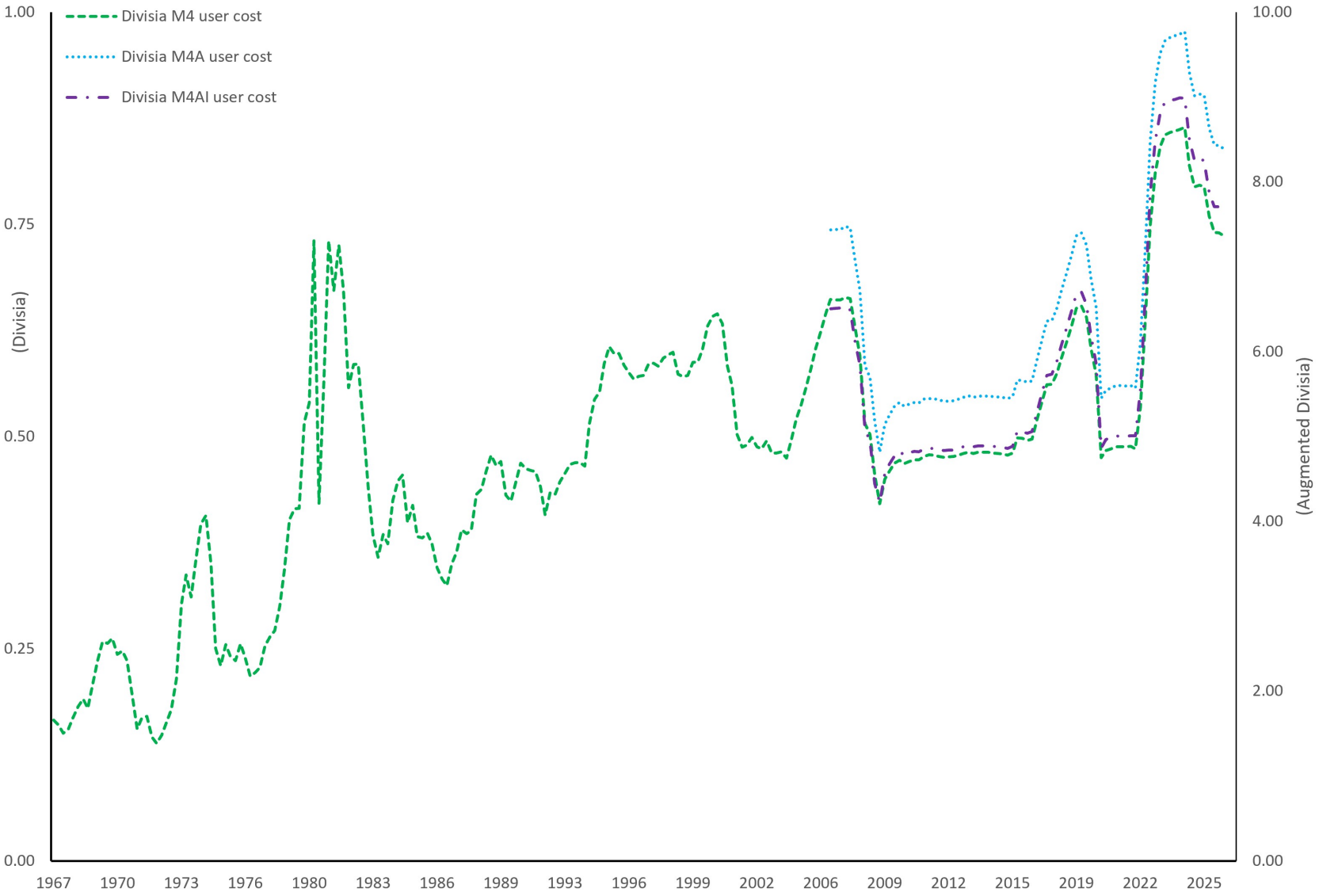


Table 1. Unit Root Tests in the Three-Variable System, {c, i, y}, 1967:q1 - 2026:q2

	Test			
Variable	ADF	DF-GLS	KPSS	Decision
		A. Real flow variables		
c	-2.310	-1.554	0.340	I(1)
i	-3.544	-2.889	0.106	I(1)
y	-2.712	-2.496	0.361	I(1)
		B. Great ratios		
c - y	-3.746	-2.365	0.114	I(1)
i - y	-3.052	-2.947	0.066	I(1)

Notes:
The 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.464, and 0.216, respectively.

Decision of I(1) indicates unit root in level found by at least two of the tests.

Table 2. Johansen ML Cointegration Tests in the Three-Variable System, {c, i, y}, 1967:q1 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
c, i, y	2	$r = 0$	0.005	0.010	1
		$r \leq 1$	0.175	0.173	
		$r \leq 2$	0.321	0.321	

Note:
Lags are selected based on the minimum SIC value of the VAR.

Table 3. Estimated Cointegration Vector(s) in the Three-Variable System, {c, i, y}, 1967:q1 - 2026:q2

Coint. vectors	System			Restrictions	p-value
	c	i	y		
1	1	0	-1	[1, 0, -1]	0.002
	0	1	-1	[0, 1, -1]	0.000

Note:
The p-values correspond to the Likelihood Ratio (LR) test for identifying restrictions.

Table 4. Unit Root Tests in the Logged Real per Capita Aggregates

Variable	Test: ADF	Test: DF-GLS	Test: KPSS	Decision
A. Simple-sum aggregates				
Sum M1	-1.226	-1.025	0.344	I(1)
Sum M2	-1.868	-1.609	0.401	I(1)
B. Original Divisia aggregates				
Divisia M1	-2.000	-0.958	0.449	I(1)
Divisia M2	-1.709	-1.136	0.484	I(1)
Divisia M3	-2.307	-1.315	0.420	I(1)
Divisia M4	-1.961	-0.965	0.379	I(1)
C. Credit-augmented Divisia aggregates				
Divisia M1A	-1.390	-1.642	0.136	I(1)
Divisia M2A	-1.615	-1.831	0.130	I(1)
Divisia M3A	-1.971	-1.961	0.145	I(1)
Divisia M4A	-1.896	-1.837	0.184	I(1)
D. Credit-augmented Divisia inside aggregates				
Divisia M1AI	-1.448	-1.667	0.128	I(1)
Divisia M2AI	-1.718	-1.899	0.122	I(1)
Divisia M3AI	-1.977	-1.971	0.154	I(1)
Divisia M4AI	-1.874	-1.735	0.184	I(1)

Notes:

For the simple-sum and original Divisia aggregates, the sample period is 1967:q1-2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.464, and 0.216, respectively.

For the credit-augmented aggregates, the sample period is 2006:q3 to 2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -4.082, -3.667, and 0.216, respectively.

Decision of I(1) indicates unit root in level found by at least two of the tests.

Table 5. Unit Root Tests in the User Costs

Variable	Test			Decision
	ADF	DF-GLS	KPSS	
A. Simple-sum aggregates				
Sum M1	-4.097	-4.110	0.047	I(0)
Sum M2	-2.023	-2.018	0.417	I(1)
B. Original Divisia aggregates				
Divisia M1	-1.950	-2.049	0.170	I(1)
Divisia M2	-3.932	-3.474	0.168	I(1)
Divisia M3	-3.968	-2.489	0.140	I(1)
Divisia M4	-2.928	-2.676	0.150	I(1)
C. Credit-augmented Divisia aggregates				
Divisia M1A	-2.548	-1.674	0.271	I(1)
Divisia M2A	-3.332	-2.130	0.201	I(1)
Divisia M3A	-3.214	-2.298	0.190	I(1)
Divisia M4A	-3.117	-2.214	0.190	I(1)
D. Credit-augmented Divisia inside aggregates				
Divisia M1AI	-2.024	-1.265	0.272	I(1)
Divisia M2AI	-3.330	-2.145	0.199	I(1)
Divisia M3AI	-3.201	-2.320	0.188	I(1)
Divisia M4AI	-3.177	-2.328	0.186	I(1)

Notes:
For the user costs of the simple-sum and original Divisia aggregates, the sample period is 1967:q1-2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.464, and 0.216, respectively.

For the user costs of the credit-augmented aggregates, the sample period is 2006:q3 to 2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -4.082, -3.667, and 0.216, respectively.

Decision of I(0) indicates stationarity found by at least two of the tests.
Decision of I(1) indicates unit root in level found by at least two of the tests.

Table 6. Johansen ML Cointegration Tests in the Money Demand System, {m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 1967:q1 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
		A. Simple-sum aggregates			
Sum M2, y, UC	3	$r = 0$	0.320	0.166	0
		$r \leq 1$	0.861	0.879	
		$r \leq 2$	0.410	0.410	
		B. Original Divisia aggregates			
Divisia M1, y, UC	3	$r = 0$	0.134	0.115	0
		$r \leq 1$	0.531	0.466	
		$r \leq 2$	0.653	0.653	
Divisia M2, y, UC	3	$r = 0$	0.009	0.005	1
		$r \leq 1$	0.426	0.426	
		$r \leq 2$	0.366	0.366	
Divisia M3, y, UC	3	$r = 0$	0.149	0.363	0
		$r \leq 1$	0.192	0.192	
		$r \leq 2$	0.192	0.192	
Divisia M4, y, UC	3	$r = 0$	0.351	0.596	0
		$r \leq 1$	0.313	0.346	
		$r \leq 2$	0.262	0.262	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 7. Estimated Cointegration Vector(s) in the Money Demand System, {m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 1967:q1 - 2026:q2

Monetary aggregate	Coint. vectors	System			Restrictions	p-value
		$m-p$	y	UC		
		A. Simple-sum aggregates				
Sum M2	0					
		B. Original Divisia aggregates				
Divisia M1	0					
Divisia M2	1	1	-1	4.460	[1,-1, UC]	0.000
Divisia M3	0					
Divisia M4	0					

Notes:
The restriction [1, -1, UC] identifies the money demand function.
The p-values correspond to the Likelihood Ratio (LR) test for identifying restrictions.

Table 8. Johansen ML Cointegration Tests in the Five-Variable System, {c, i, m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 1967:q1 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
A. Simple-sum aggregates					
c, i, Sum M2, y, UC	2	$r = 0$	0.002	0.004	1
		$r \leq 1$	0.149	0.062	
		$r \leq 2$	0.44	0.615	
		$r \leq 3$	0.886	0.885	
		$r \leq 4$	0.497	0.497	
B. Original Divisia aggregates					
c, i, Divisia M1, y, UC	3	$r = 0$	0.002	0.007	1
		$r \leq 1$	0.114	0.281	
		$r \leq 2$	0.252	0.339	
		$r \leq 3$	0.409	0.326	
		$r \leq 4$	0.999	0.999	
c, i, Divisia M2, y, UC	3	$r = 0$	0.000	0.000	2
		$r \leq 1$	0.038	0.030	
		$r \leq 2$	0.436	0.664	
		$r \leq 3$	0.368	0.296	
		$r \leq 4$	0.760	0.760	
c, i, Divisia M3, y, UC	3	$r = 0$	0.000	0.000	1
		$r \leq 1$	0.051	0.048	
		$r \leq 2$	0.420	0.546	
		$r \leq 3$	0.462	0.459	
		$r \leq 4$	0.378	0.378	
c, i, Divisia M4, y, UC	2	$r = 0$	0.003	0.004	1
		$r \leq 1$	0.165	0.105	
		$r \leq 2$	0.652	0.890	
		$r \leq 3$	0.402	0.449	
		$r \leq 4$	0.261	0.261	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 9. Estimated Cointegration Vector(s) in the Five-Variable System, {c, i, m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 1967:q1 - 2026:q2

Monetary aggregate	Coint. vectors	System c	i	$m-p$	y	UC	Restrictions	p-value
				A. Simple-sum aggregates				
Sum M2	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.590	[0, 0, 1, -1, UC]	0.000
				B. Original Divisia aggregates				
Divisia M1	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	5.532	[0, 0, 1, -1, UC]	0.000
Divisia M2	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.012
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	5.223	[0, 0, 1, -1, UC]	0.008
Divisia M3	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	3.018	[0, 0, 1, -1, UC]	0.000
Divisia M4	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	3.567	[0, 0, 1, -1, UC]	0.000

Notes:
The restrictions [1, 0, 0, -1, 0], [0, 1, 0, -1, 0], and [0, 0, 1, -1, UC] identify the consumption-output great ratio, investment-output great ratio, and the money demand function, respectively.
The p-values correspond to the Likelihood Ratio (LR) test for identifying restrictions. and
proper user costs as the opportunity cost.

Table 10. Johansen ML Cointegration Tests in the Three-Variable Money Demand System, {m-p, y, UC}, with the Credit-Augmented Aggregates, 2006:q3 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
A. Credit-augmented Divisia aggregates					
Divisia M1A, y, UC	3	$r = 0$	0.007	0.006	1
		$r \leq 1$	0.327	0.255	
		$r \leq 2$	0.856	0.856	
Divisia M2A, y, UC	2	$r = 0$	0.001	0.000	1
		$r \leq 1$	0.514	0.427	
		$r \leq 2$	0.901	0.901	
Divisia M3A, y, UC	2	$r = 0$	0.014	0.013	1
		$r \leq 1$	0.338	0.267	
		$r \leq 2$	0.789	0.789	
Divisia M4A, y, UC	3	$r = 0$	0.001	0.015	1
		$r \leq 1$	0.227	0.179	
		$r \leq 2$	0.640	0.640	
B. Credit-augmented Divisia inside aggregates					
Divisia M1AI, y, UC	3	$r = 0$	0.021	0.020	1
		$r \leq 1$	0.358	0.288	
		$r \leq 2$	0.747	0.747	
Divisia M2AI, y, UC	3	$r = 0$	0.001	0.000	1
		$r \leq 1$	0.547	0.458	
		$r \leq 2$	0.963	0.963	
Divisia M3AI, y, UC	2	$r = 0$	0.172	0.211	0
		$r \leq 1$	0.417	0.335	
		$r \leq 2$	0.891	0.891	
Divisia M4AI, y, UC	2	$r = 0$	0.136	0.205	0
		$r \leq 1$	0.328	0.253	
		$r \leq 2$	0.968	0.968	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 11. Estimated Cointegration Vector(s) in the Money Demand System, {m-p, y, UC}, with the Credit-Augmented Aggregates, 2006:q3 - 2026:q2

Monetary aggregate	Coint. vectors	System			Restrictions	p-value
		$m-p$	y	UC		
A. Credit-augmented Divisia aggregates						
Divisia M1A	1	1	-1	0.212	[1,-1, UC]	0.008
Divisia M2A	1	1	-1	0.274	[1,-1, UC]	0.677
Divisia M3A	1	1	-1	0.074	[1,-1, UC]	0.274
Divisia M4A	1	1	-1	0.079	[1,-1, UC]	0.038
B. Credit-augmented Divisia inside aggregates						
Divisia M1AI	1	1	-1	0.259	[1,-1, UC]	0.006
Divisia M2AI	1	1	-1	0.258	[1,-1, UC]	0.728
Divisia M3AI	0					
Divisia M4AI	0					

Notes:
The restriction [1, -1, UC] identifies the money demand function.
The p-values correspond to the Likelihood Ratio (LR) test for identifying restrictions.

Table 12. Johansen ML Cointegration Tests in the Five-Variable System, {c, i, m-p, y, UC}, with the Credit-Augmented Aggregates, 2006:q3 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
A. Credit-augmented Divisia aggregates					
c, i, Divisia M1A, y, UC	3	$r = 0$	0.000	0.005	2
		$r \leq 1$	0.005	0.009	
		$r \leq 2$	0.167	0.122	
		$r \leq 3$	0.620	0.654	
		$r \leq 4$	0.327	0.327	
c, i, Divisia M2A, y, UC	3	$r = 0$	0.000	0.015	2
		$r \leq 1$	0.009	0.011	
		$r \leq 2$	0.252	0.570	
		$r \leq 3$	0.205	0.359	
		$r \leq 4$	0.088	0.088	
c, i, Divisia M3A, y, UC	3	$r = 0$	0.000	0.028	2
		$r \leq 1$	0.000	0.005	
		$r \leq 2$	0.060	0.062	
		$r \leq 3$	0.399	0.318	
		$r \leq 4$	0.914	0.914	
c, i, Divisia M4A, y, UC	3	$r = 0$	0.000	0.026	2
		$r \leq 1$	0.003	0.043	
		$r \leq 2$	0.033	0.049	
		$r \leq 3$	0.272	0.222	
		$r \leq 4$	0.608	0.608	
B. Credit-augmented Divisia inside aggregates					
c, i, Divisia M1AI, y, UC	3	$r = 0$	0.000	0.006	2
		$r \leq 1$	0.004	0.008	
		$r \leq 2$	0.182	0.135	
		$r \leq 3$	0.628	0.678	
		$r \leq 4$	0.301	0.301	
c, i, Divisia M2AI, y, UC	3	$r = 0$	0.000	0.015	2
		$r \leq 1$	0.010	0.011	
		$r \leq 2$	0.290	0.595	
		$r \leq 3$	0.235	0.371	
		$r \leq 4$	0.110	0.110	
c, i, Divisia M3AI, y, UC	3	$r = 0$	0.000	0.026	2
		$r \leq 1$	0.000	0.004	
		$r \leq 2$	0.056	0.057	
		$r \leq 3$	0.400	0.321	
		$r \leq 4$	0.834	0.834	
c, i, Divisia M4AI, y, UC	3	$r = 0$	0.000	0.021	2
		$r \leq 1$	0.001	0.006	
		$r \leq 2$	0.066	0.044	
		$r \leq 3$	0.555	0.479	
		$r \leq 4$	0.742	0.742	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 13. Estimated Cointegration Vector(s) in the Five-Variable System, {c, i, m-p, y, UC}, with the Credit-Augmented Aggregates, 2006:q3 - 2026:q2

Monetary aggregate	Coint. vectors	System c	i	$m-p$	y	UC	Restrictions	p-value
		A. Credit-augmented Divisia aggregates						
Divisia M1A	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.152	[0, 0, 1, -1, UC]	0.180
Divisia M2A	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.209	[0, 0, 1, -1, UC]	0.999
Divisia M3A	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.052	[0, 0, 1, -1, UC]	0.113
Divisia M4A	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.002
		0.000	0.000	1.000	-1.000	0.062	[0, 0, 1, -1, UC]	0.173
		B. Credit-augmented Divisia aggregates inside						
Divisia M1AI	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.177	[0, 0, 1, -1, UC]	0.073
Divisia M2AI	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.217	[0, 0, 1, -1, UC]	0.999
Divisia M3AI	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.054	[0, 0, 1, -1, UC]	0.068
Divisia M4AI	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.001
		0.000	0.000	1.000	-1.000	0.044	[0, 0, 1, -1, UC]	0.032

Notes:
The restrictions [1, 0, 0, -1, 0], [0, 1, 0, -1, 0], and [0, 0, 1, -1, UC] identify the consumption-output great ratio, investment-output great ratio, and the money demand function, respectively.
The p-values correspond to the Likelihood Ratio (LR) tests for identifying restrictions.

Table 14. Johansen ML Cointegration Tests in the Money Demand System, {m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 2006:q3 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
A. Simple-sum aggregates					
Sum M2, y, UC	3	$r = 0$	0.139	0.126	0
		$r \leq 1$	0.516	0.428	
		$r \leq 2$	0.999	0.999	
B. Original Divisia aggregates					
Divisia M1, y, UC	3	$r = 0$	0.006	0.005	1
		$r \leq 1$	0.326	0.251	
		$r \leq 2$	0.997	0.997	
Divisia M2, y, UC	3	$r = 0$	0.056	0.032	0
		$r \leq 1$	0.596	0.521	
		$r \leq 2$	0.745	0.745	
Divisia M3, y, UC	2	$r = 0$	0.223	0.229	0
		$r \leq 1$	0.514	0.429	
		$r \leq 2$	0.873	0.873	
Divisia M4, y, UC	2	$r = 0$	0.037	0.169	0
		$r \leq 1$	0.087	0.058	
		$r \leq 2$	0.959	0.959	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 15. Estimated Cointegration Vector(s) in the Money Demand System, {m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 2006:q3 - 2026:q2

Monetary aggregate	Coint. vectors	System			Restrictions	p-value
		$m-p$	y	UC		
		A. Simple-sum aggregates				
Sum M2	0					
		B. Original Divisia aggregates				
Divisia M1	1	1	-1	2.597	[1,-1, UC]	0.007
Divisia M2	0					
Divisia M3	0					
Divisia M4	0					

Notes:
The restriction [1, -1, UC] identifies the money demand function.
The p-values correspond to the Likelihood Ratio (LR) tests for identifying restrictions.

Table 16. Johansen ML Cointegration Tests in the Five-Variable System, {c, i, m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 2006:q3 - 2026:q2

System	Lag	Null	Cointegration tests λ_{trace}	Cointegration tests λ_{max}	Coint. vectors
A. Simple-sum aggregates					
c, i, Sum M2, y, UC	3	$r = 0$	0.001	0.001	1
		$r \leq 1$	0.150	0.346	
		$r \leq 2$	0.278	0.346	
		$r \leq 3$	0.449	0.379	
		$r \leq 4$	0.691	0.691	
B. Original Divisia aggregates					
c, i, Divisia M1, y, UC	3	$r = 0$	0.000	0.006	1
		$r \leq 1$	0.033	0.100	
		$r \leq 2$	0.177	0.127	
		$r \leq 3$	0.635	0.639	
		$r \leq 4$	0.401	0.401	
c, i, Divisia M2, y, UC	3	$r = 0$	0.002	0.011	1
		$r \leq 1$	0.077	0.277	
		$r \leq 2$	0.168	0.538	
		$r \leq 3$	0.126	0.210	
		$r \leq 4$	0.102	0.102	
c, i, Divisia M3, y, UC	3	$r = 0$	0.000	0.013	1
		$r \leq 1$	0.011	0.128	
		$r \leq 2$	0.048	0.052	
		$r \leq 3$	0.370	0.292	
		$r \leq 4$	0.895	0.895	
c, i, Divisia M4, y, UC	3	$r = 0$	0.001	0.072	2
		$r \leq 1$	0.005	0.044	
		$r \leq 2$	0.062	0.110	
		$r \leq 3$	0.248	0.200	
		$r \leq 4$	0.603	0.603	

Note:
Lags are selected using the minimum SIC value for the VAR and adjusted as needed to ensure that the residuals are free from serial correlation.

Table 17. Estimated Cointegration Vector(s) in the Five-Variable System, {c, i, m-p, y, UC}, with the Simple-Sum and Original Divisia Aggregates, 2006:q3 - 2026:q2

Monetary aggregate	Coint. vectors	System c	i	$m-p$	y	UC	Restrictions	p-value
				A. Simple-sum aggregates				
Sum M2	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.839	[0, 0, 1, -1, UC]	0.000
				B. Original Divisia aggregates				
Divisia M1	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	2.064	[0, 0, 1, -1, UC]	0.000
Divisia M2	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	2.149	[0, 0, 1, -1, UC]	0.001
Divisia M3	1	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.003
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.000
		0.000	0.000	1.000	-1.000	0.557	[0, 0, 1, -1, UC]	0.000
Divisia M4	2	1.000	0.000	0.000	-1.000	0.000	[1, 0, 0, -1, 0]	0.000
		0.000	1.000	0.000	-1.000	0.000	[0, 1, 0, -1, 0]	0.001
		0.000	0.000	1.000	-1.000	0.773	[0, 0, 1, -1, UC]	0.146

Notes:
The restrictions [1, 0, 0, -1, 0], [0, 1, 0, -1, 0], and [0, 0, 1, -1, R] identify the consumption-output great ratio, investment-output great ratio, and the money demand function, respectively.
The p-values correspond to the Likelihood Ratio (LR) tests for identifying restrictions.

Table 18. Comparison of cointegration test results, 1967:q1 - 2026:q2

	System			
	{m-p, y, UC}		{c, i, m-p, y, UC}	
Aggregates	Coint. vectors	Restriction not rejected	Coint. vectors	Restriction(s) not rejected
Sum M2	0	-	1	-
Divisia M1	0	-	1	-
Divisia M2	1	-	2	-
Divisia M3	0	-	1	-
Divisia M4	0	-	1	-

Notes:
In addition to the two systems, we estimated another three-variable system, {c, i, y}, and found 1 cointegrating vector. The tested restrictions are [1, 0, -1] and [0, 1, -1], which are rejected.
For the three-variable system, {m-p, y, UC}, the tested restriction is [1, -1, UC].
For the five-variable system, {c, i, m-p, y, UC}, the tested restrictions are [1, 0, 0, -1, 0], [0, 1, 0, -1, 0], and [0, 0, 1, -1, UC].

Table 19. Comparison of cointegration test results, 2006:q3 - 2026:q2

	System			
	{m-p, y, UC}		{c, i, m-p, y, UC}	
Aggregates	Coint. vectors	Restriction not rejected	Coint. vectors	Restriction(s) not rejected
Sum M2	0	-	1	-
Divisia M1	1	-	1	-
Divisia M2	0	-	1	-
Divisia M3	0	-	1	-
Divisia M4	0	-	2	[0, 0, 1, -1, UC]
Divisia M1A	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M2A	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M3A	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M4A	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M1AI	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M2AI	1	[1, -1, UC]	2	[0, 0, 1, -1, UC]
Divisia M3AI	0	-	2	[0, 0, 1, -1, UC]
Divisia M4AI	0	-	2	-

Notes:
For the three-variable system, {m-p, y, UC}, the tested restriction is [1, -1, UC].
For the five-variable system, {c, i, m-p, y, UC}, the tested restrictions are [1, 0, 0, -1, 0], [0, 1, 0, -1, 0], and [0, 0, 1, -1, UC].

Appendix Table A1. Unit Root Tests in the Three-Variable System, {c, i, y}, 1967:q1 - 2026:q2

Variable	Test			
(First Difference)	ADF	DF-GLS	KPSS	Decision
		A. Real flow variables		
c	-16.461	-15.862	0.042	I(0)
i	-8.921	-8.530	0.029	I(0)
y	-15.490	-14.386	0.022	I(0)
		B. Great ratios		
c - y	-16.104	-4.504	0.037	I(0)
i - y	-6.828	-5.442	0.035	I(0)

Notes:
The 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.4635, and 0.216, respectively.
Decision of I(0) indicates stationarity found by at least two of the tests.

Appendix Table A2. Unit Root Tests in the Logged Real per Capita Aggregates

Variable	Test			
(First Difference)	ADF	DF-GLS	KPSS	Decision
		A. Simple-sum aggregates		
Sum M1	-11.528	-11.504	0.053	I(0)
Sum M2	-8.152	-4.532	0.053	I(0)
		B. Original Divisia aggregates		
Divisia M1	-8.849	-6.369	0.053	I(0)
Divisia M2	-4.734	-4.229	0.073	I(0)
Divisia M3	-4.876	-4.346	0.066	I(0)
Divisia M4	-9.899	-9.453	0.055	I(0)
		C. Credit-augmented Divisia aggregates		
Divisia M1A	-3.545	-3.371	0.108	I(1)
Divisia M2A	-3.430	-3.438	0.082	I(1)
Divisia M3A	-4.344	-4.327	0.092	I(0)
Divisia M4A	-5.500	-5.489	0.085	I(0)
		D. Credit-augmented Divisia inside aggregates		
Divisia M1AI	-3.750	-3.549	0.106	I(1)
Divisia M2AI	-3.442	-3.463	0.077	I(1)
Divisia M3AI	-4.348	-4.299	0.086	I(0)
Divisia M4AI	-4.431	-4.266	0.084	I(0)

Notes:
For the simple-sum and original Divisia aggregates, the sample period is 1967:q1-2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.464, and 0.216, respectively.

For the credit-augmented aggregates, the sample period is 2006:q3 to 2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -4.078, -3.660, and 0.216, respectively.

Decision of I(0) indicates stationarity found by at least two of the tests.
Decision of I(1) indicates unit root in level found by at least two of the tests.

Appendix Table A3. Unit Root Tests in the User Costs

Variable	Test			
(First Difference)	ADF	DF-GLS	KPSS	Decision
		A. Simple-sum aggregates		
Sum M1	-13.261	-4.554	0.022	I(0)
Sum M2	-18.008	-18.078	0.057	I(0)
		B. Original Divisia aggregates		
Divisia M1	-11.042	-10.605	0.054	I(0)
Divisia M2	-12.711	-12.086	0.031	I(0)
Divisia M3	-13.945	-13.352	0.032	I(0)
Divisia M4	-14.919	-14.405	0.033	I(0)
		C. Credit-augmented Divisia aggregates		
Divisia M1A	-3.858	-3.911	0.053	I(0)
Divisia M2A	-5.929	-5.847	0.059	I(0)
Divisia M3A	-7.001	-6.708	0.049	I(0)
Divisia M4A	-7.700	-7.248	0.046	I(0)
		D. Credit-augmented Divisia inside aggregates		
Divisia M1AI	-3.534	-3.594	0.053	I(1)
Divisia M2AI	-5.982	-5.883	0.058	I(0)
Divisia M3AI	-7.157	-6.815	0.051	I(0)
Divisia M4AI	-7.419	-6.996	0.049	I(0)

Notes:
For the user costs of the simple-sum and original Divisia aggregates, the sample period is 1967:q1-2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -3.998, -3.464, and 0.216, respectively.

For the user costs of the credit-augmented aggregates, the sample period is 2006:q3 to 2026:q2, and the 1 percent critical values for the ADF, DF-GLS, and KPSS tests are -4.078, -3.660, and 0.216, respectively.

Decision of I(0) indicates stationarity found by at least two of the tests.
Decision of I(1) indicates unit root in level found by at least two of the tests.

Appendix A

Aggregation-Theoretic Measures of Monetary Services

This appendix summarizes the theoretical foundations of the monetary aggregates used in the empirical analysis. The measures differ according to how they aggregate the transaction services generated by monetary assets. The simple-sum aggregates assign identical weights to all monetary assets, whereas the Divisia family of monetary aggregates weights assets according to the monetary services they provide. The credit-augmented measures further recognize that modern payment systems generate transaction services through both monetary assets and credit cards.

Simple-Sum Monetary Aggregates

Most central banks publish monetary aggregates constructed using the simple-sum index. Under this approach, the aggregate equals the arithmetic sum of its component assets,

$$M_t = \sum_{i=1}^{I} x_{it} \tag{A1}$$

where x_{it} denotes the quantity of monetary asset i.

This aggregation procedure assigns every component asset an identical weight regardless of differences in liquidity, transaction services, or rates of return. Consequently, it implicitly assumes that all monetary assets are perfect substitutes in providing monetary services.

User Cost of Monetary Assets

Aggregation theory begins by recognizing that economic agents allocate wealth between monetary assets and a benchmark asset that yields the highest available return but provides no transaction services. The opportunity cost of holding a monetary asset instead of the benchmark asset represents the value of the monetary services supplied by that asset.

Following Barnett (1978), the real user cost of monetary asset i is

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t} \tag{A2}$$

where R_t denotes the benchmark rate of return and r_{it} denotes the own rate of return on monetary asset i.

Assets with lower own rates of return have larger user costs because holders sacrifice more investment income in exchange for transaction services.

Divisia Monetary Aggregates

The Divisia monetary aggregate measures the growth rate of aggregate monetary services as a weighted average of the growth rates of the component assets. Unlike

the simple-sum index, the weights vary over time according to the expenditure shares implied by user costs.

The growth rate of the Divisia monetary aggregate is

$$d \ln M_t = \sum_{i=1}^{I} s_{it} d \ln m_{it} \tag{A3}$$

where the expenditure share of monetary asset i is

$$s_{it} = \frac{\pi_{it} m_{it}}{\sum_{i=1}^{I} \pi_{it} m_{it}} \tag{A4}$$

and m_{it} denotes the real quantity of monetary asset i.

This aggregation procedure allows each monetary asset to contribute according to the monetary services it provides instead of assigning identical weights to all assets.

Credit-Augmented Divisia Monetary Aggregates

Traditional monetary aggregates do not account for the deferred-payment services generated by credit cards. Since these services facilitate transactions in much the same way as conventional monetary assets, excluding them omits an important component of monetary services. Under aggregation theory, the user cost associated with credit-card transaction service l is

$$\pi_{lt} = \frac{e_{lt} - R_t}{1 + R_t} \tag{A5}$$

where e_{lt} denotes the expected borrowing cost associated with credit-card transaction l.

Including both monetary assets and credit-card services yields the credit-augmented Divisia monetary aggregate

$$d \ln M_t = \sum_{i=1}^{I} s_{it} d \ln m_{it} + \sum_{l=1}^{L} s_{lt} d \ln m_{lt} \tag{A6}$$

where the expenditure shares become

$$s_{it} = \frac{\pi_{it} m_{it}}{\sum_{i=1}^{I} \pi_{it} m_{it} + \sum_{l=1}^{L} \pi_{lt} m_{lt}} \tag{A7}$$

and

$$s_{lt} = \frac{\pi_{lt} m_{lt}}{\sum_{i=1}^{I} \pi_{it} m_{it} + \sum_{l=1}^{L} \pi_{lt} m_{lt}} \tag{A8}$$

This measure jointly aggregates the transaction services generated by conventional monetary assets and credit cards within a single theoretically consistent framework.

Credit-Augmented Divisia Inside Monetary Aggregates

The previous aggregates measure the monetary services demanded by households and firms. An alternative approach measures the monetary services supplied by financial intermediaries. In this framework, privately produced inside money and credit-card services are aggregated from the supply side. The corresponding user cost received by financial intermediaries is

$$\pi_{it} = \frac{(1 - k_i)R_t - r_{it}}{1 + R_t} \tag{A9}$$

where k_i denotes the reserve requirement applicable to monetary asset i.

The resulting credit-augmented Divisia inside aggregates are constructed using the same Divisia index methodology as before, replacing quantities demanded by quantities supplied and replacing household user costs with the corresponding supply-side user costs.

Unlike the conventional Divisia aggregates, the inside aggregates exclude outside money, such as currency, and instead focus on privately supplied monetary assets together with credit-card transaction services.

The monetary aggregates employed in this study differ only in the method used to aggregate monetary services. Simple-sum aggregates assign equal weights to all component assets. Divisia aggregates replace these fixed weights with expenditure shares derived from user costs. Credit-augmented Divisia aggregates further incorporate the transaction services generated by credit cards, while credit-augmented Divisia inside aggregates measure the monetary services supplied by financial intermediaries rather than those demanded by asset holders. These progressively richer measures provide an increasingly comprehensive representation of monetary services and form the basis of the empirical analysis in this paper.